

TECHNICAL APPENDIX TO

Detection Avoidance, 81 NYU L. REV. __ (2006)

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This appendix reformulates in mathematical terms arguments made verbally in Detection Avoidance, 81 NYU L. REV. __ (2006).

1. Model

Let a_i denote spending on detection avoidance of “order” i . Thus, a_1 is spending to avoid detection of the underlying violation, a_2 is spending to avoid detection of activities attendant to a_1 , etc... Let d_i represent public spending devoted to detecting avoidance of order i , incurred in proportion to the extent to which perpetrators engage in order i avoidance.¹ The variable d_0 is detection effort directed at the underlying violation.² Let s_i be the sanction imposed on detection avoidance of order i . The variable s_0 is the sanction imposed on the underlying violation. We retain three standard assumptions from the neoclassical approach (as described in Part 2.1): that offenders are risk neutral; that they are rational; and that there are no false positives in detection.

In a model without detection avoidance the level of deterrence (of the underlying activity) is: $\Delta^0 = p_0(d_0)s_0$, where $p_0(d_0)$ is the probability of detection for the underlying activity, a function of respective detection effort. This expected sanction conditional on committing the violation is what the potential violator weighs against her private gains from the illicit activity in deciding whether to commit the offense.

With detection avoidance added to the model, the level of deterrence for the underlying violation is the violator’s *minimized all-in* expected sanction conditional on commission of the underlying violation, where the violator minimizes this expected sanction by choosing the sequence a_1, a_2, \dots

¹ The model can also accommodate “fixed” costs of both detection and detection avoidance.

² We could also include a fixed cost component to detection, incurred irrespective of the number of violations. Symmetrically, we could also include a fixed cost component for detection avoidance, incurred regardless of whether a violation is committed.

$$\begin{aligned}
 \Delta = \min_{a_1, a_2, \dots} & \underbrace{p_0 \left(\overset{+}{d}_0; \overset{-}{a}_1, \overset{-}{a}_2, \dots \right)}_{\text{expected sanction for underlying violation}} s_0 \\
 & + \sum_{i=1}^{\infty} \underbrace{a_i}_{\substack{\text{spending} \\ \text{on } i\text{th} \\ \text{order} \\ \text{detection} \\ \text{avoidance}}} + \sum_{i=1}^{\infty} \underbrace{p_i \left(\overset{+}{d}_i, \overset{+}{a}_i; \overset{-}{a}_{i+1}, \overset{-}{a}_{i+2}, \dots \right)}_{\text{expected sanction for } i\text{th order detection avoidance}} s_i
 \end{aligned} \tag{1}$$

In this expression $p_i(d_i, a_i; a_{i+1}, a_{i+2}, \dots)$ is the probability of detection for i^{th} order detection avoidance, a function of i^{th} order detection effort, i^{th} order detection avoidance itself, and all higher orders of detection avoidance. Assumed signs of first derivatives are indicated above the respective arguments. Three things are worth noticing about this expression. First, the probability of detection for i^{th} order detection avoidance (including 0^{th} order, the primary activity) is a positive function of detection avoidance of order i and a negative function of detection avoidance of orders $i+1, i+2, \text{ etc.} \dots$. Consider for example, $i = 1$. The more the violator covers up, the more likely that cover up is detected. The more the violator covers up the cover up, however, the less likely that first order cover up is detected. Moreover, detection of *first* order cover up is also rendered less likely, the more the violator engages in *third* order cover up—i.e., covers up the cover up of cover up. This last effect is not emphasized in the text, but is added here for realism. The idea is that higher orders of detection avoidance shore up all lower orders.

Second, no assumption is made here regarding the statistic dependence or independence of detection of detection avoidance of order i relative to detection of detection avoidance of any other order $j \neq i$. The expectation operator is linear, regardless of statistical dependence: i.e., $E[X + Y] = E[X] + E[Y]$ even if X and Y are statistically dependent.

Third, and most importantly, we see in expression (1) the formal manifestation of the three effects of detection avoidance on primary activity deterrence, as described verbally in Part 4.1. First, detection avoidance (of all orders) lowers the probability of detection for the underlying violation. This is signified by the fact that increasing any or all coordinates of the sequence a_1, a_2, \dots reduces p_0 . Second, this reduction is paid for by the violator and that payment adds to the all-in effective sanction: detection avoidance spending a_i directly increases deterrence, as shown in

the first summation in (1). Lastly, expected sanctions for detection avoidance (s_1, s_2, \dots) contribute to the all-in effective sanction for the *underlying* violation.

Violators whose private benefits exceed deterrence choose to commit the offense. If private benefits are distributed according to the cumulative distribution F , therefore, $1 - F(\Delta)$ potential offenders choose to commit the offense. Social costs from producing deterrence level Δ are thus:

$$C(\Delta) = \underbrace{(1 - F(\Delta))}_{\text{proportion of offenders}} \left(\sum_{i=1}^{\infty} a_i + \sum_{i=0}^{\infty} d_i + \sum_{i=0}^{\infty} \underbrace{p_i c s_i}_{\text{cost of sanctions}} \right)$$

We see here the formal manifestation of the new social costs borne from adding detection avoidance to the model, as described in Part 4.2. The first sum captures the social waste of detection avoidance. The second, the public cost of detecting not just the primary violation, but also detection avoidance. And the third sum represents the cost of sanctioning not just detected primary violations but also detected detection avoidance activities. Following Becker, the coefficient c is the cost per sanction imposed. To simplify the analysis, assume from hereon that $c = 0$. Such is the case when the sanction is a fine.

Letting $B(\Delta)$ represent the social benefits of deterrence, the social problem is to choose s_0, s_1, s_2, \dots and d_0, d_1, d_2, \dots to maximize $B(\Delta) - C(\Delta)$. The preliminary, social cost minimization problem that we focus on is choice of the most efficient means of producing any given level of deterrence $\bar{\Delta}$: $\min_{s_i, d_i} C(\bar{\Delta}) : \Delta = \bar{\Delta}$.

The marginal efficiency of a policy instrument x will be measured by its marginal deterrence cost: its marginal contribution to costs per its marginal contribution to deterrence:

$$MDC_x \equiv \frac{\delta C}{\delta x} / \frac{\delta \Delta}{\delta x}.$$

The greater this ratio, the less efficient is instrument x on the margin. At a solution to the social cost minimization problem given $\bar{\Delta}$, the marginal deterrence cost of all instruments positively employed must be equal. Otherwise, the cost of producing deterrence $\bar{\Delta}$ could be decreased by substituting low marginal

deterrence cost instruments for high. Roughly speaking, instruments whose marginal deterrence cost tends to be lower (across all levels of their own employment and the employment of other instruments) will tend to be more heavily employed at an optimum. If society is currently not at a social optimum, then the marginal deterrence costs of instruments may not be equal. In that case, the most efficient way to increase deterrence on the margin is to rely on those instruments with the lowest marginal deterrence costs. Furthermore, holding the level of deterrence constant, such deterrence can be more efficiently provided by substituting low marginal deterrence cost instrument for high.

From (1) we may derive the first order condition for an optimum in the violator's detection avoidance choice problem. We will do so in a way that corresponds to the text discussion of the upside and downside of each order of detection avoidance. (See, e.g., Part 6.1.1.) The (marginal) "sanctions-decreasing upside" of first order detection avoidance is the degree to which it lowers the expected sanction for the primary violation: $-\frac{\delta p_0}{\delta a_1} s_0 > 0$. We will assume that the derivative in this expression is strictly decreasing (in an absolute value sense) in the level of first order detection avoidance. In other words, first order detection avoidance spending has a diminishing marginal impact on the probability of detection for the underlying violation. In fact, we will assume this generally for the impact of higher order detection avoidance on lower order detection probabilities. Against this marginal upside the violator weighs two marginal costs: the direct marginal cost, which is simply one; and the (marginal) "sanctions-increasing downside:" $\frac{\delta p_1}{\delta a_1} s_1 > 0$. We assume that the derivative in this expression is strictly increasing in the level of first order detection avoidance: spending on first order detection avoidance increases the chance of detection thereof at an increasing rate. The violator sets first order detection avoidance at the point where its marginal productivity equals its marginal cost: $-\frac{\delta p_0}{\delta a_1} s_0 = \frac{\delta p_1}{\delta a_1} s_1 + 1$. (If this equation cannot be satisfied, the violator is at a corner solution, $a_1 = 0$.) More generally, the sanctions-reducing upside for detection avoidance of order i is $-\sum_{j=0}^{i-1} \frac{\delta p_j}{\delta a_i} s_j > 0$. The direct cost and sanctions-increasing downside are the same as for first order detection avoidance, and the violator sets: $-\sum_{j=0}^{i-1} \frac{\delta p_j}{\delta a_i} s_j = \frac{\delta p_i}{\delta a_i} s_i + 1$, unless this cannot be satisfied in which case $a_i = 0$.

2. Sanctioning underlying violations

Suppose for purposes of illustration that detection avoidance is not sanctioned: $s_1 = s_2 = \dots = 0$. In that case, the avoider sets each a_i according to: $-\frac{\delta p_0}{\delta a_i} s_0 = 1$, assuming an interior solution at all orders. Increasing the sanction s_0 for the primary violation raises the sanctions-decreasing upside for first order avoidance as well as avoidance of all higher orders. How does this affect detection avoidance at each order?

To render this difficult problem tractable assume that cross effects across orders of detection avoidance are negligible: $\frac{\delta^2 p_0}{\delta a_i \delta a_j} = 0$, $i \neq j$. (For a discussion of such cross effects see Part 5.2.) In this case the equations $-\frac{\delta p_0}{\delta a_i} s_0 = 1$, for $i = 1, 2, \dots$ are independent and for each i the implicit function theorem yields:

$$\frac{\delta a_i}{\delta s_0} = - \left(\frac{\overbrace{\delta p_0}^+}{\delta a_i} / \underbrace{-\frac{\delta^2 p_0}{\delta a_i \delta a_i} s_0}_{-} \right) > 0.$$

Therefore, increasing the primary sanction increases detection avoidance of each order. Raising the sanction raises the marginal sanction-reducing downside of each order of detection avoidance. The violator increases each order of detection avoidance until the marginal upside diminishes back down to the marginal direct cost, 1.

On the other hand, increasing s_0 does increase primary deterrence. Applying the envelope theorem:

$$\frac{\delta \Delta}{\delta s_0} = p_0 + \underbrace{\sum_{i=1}^{\infty} \left(\frac{\delta p_0}{\delta a_i} s_0 + 1 \right)}_{=0} \frac{\delta a_i}{\delta s_0} = p_0 > 0. \quad (2)$$

Notice that the term above the horizontal bracket zeros out even for a_i that are at the corner, $a_i = 0$. Although $\frac{\delta p_0}{\delta a_i} s_0 + 1$ need not be zero at a corner, if it is not, then $\frac{\delta a_i}{\delta s_0}$ will be.

Therefore, increasing the sanction for the primary violation increases deterrence of the primary violation by p_0 , but also increases detection avoidance. Thus, even though the sanction is

costless to impose ($c = 0$), the deterrence it generates comes at a social cost, and its marginal cost per deterrence is positive not zero, as in Becker:

$$\frac{\delta C}{\delta s_0} \bigg/ \frac{\delta \Delta}{\delta s_0} = (1 - F(\bar{\Delta})) \sum_{i=1}^{\infty} \frac{\delta a_i}{\delta s_0} \bigg/ p_0 > 0.$$

This generalizes Malik's result, as described in Part 2.2.

3. Sanctioning detection avoidance

The effect on higher orders of detection avoidance of imposing a sanction s_1 on first order detection avoidance is similar in form to the effect on all orders of detection avoidance of raising the sanction s_0 on the primary violation. First order detection avoidance plays the role played by the primary violation. For higher order detection avoidance a_2, a_3, \dots the offender sets: $-\frac{\delta p_0}{\delta a_i} s_0 - \frac{\delta p_1}{\delta a_i} s_1 = 1$. By the implicit function theorem, the effect of raising s_1 is

$$\frac{\delta a_i}{\delta s_1} = - \left(\frac{\overbrace{\delta p_1}^{+}}{-\delta a_i} \bigg/ \underbrace{-\frac{\delta^2 p_1}{\delta a_i \delta a_i} s_1}_{-} \right) > 0, \quad i \geq 2$$

Similarly, raising the sanction on i^{th} order detection avoidance increases detection avoidance at all orders higher than i :

$$\frac{\delta a_{i+k}}{\delta s_i} = - \left(\frac{\overbrace{\delta p_i}^{+}}{-\delta a_{i+k}} \bigg/ \underbrace{-\frac{\delta^2 p_i}{\delta a_{i+k} \delta a_{i+k}} s_i}_{-} \right) > 0, \quad k \geq 1.$$

The effect on primary deterrence of raising the sanction on i^{th} order detection avoidance is also simplified by the envelope theorem, to p_i . Raising the sanction on i^{th} order detection avoidance (keeping the sanction on lower order detection avoidance fixed) has marginal deterrence cost:

$$\frac{\delta C}{\delta s_i} \bigg/ \frac{\delta \Delta}{\delta s_i} = (1 - F(\bar{\Delta})) \frac{\overbrace{\delta a_i}^{-} + \sum_{k=1}^{\infty} \overbrace{\delta a_{i+k}}^{+}}{\delta s_i} \bigg/ p_i.$$

This cost may be positive or negative depending on relative responses at i^{th} and higher orders of detection avoidance.

4. Sanctioning hierarchies: uniform

Suppose that we impose a uniform sanction s on all orders of detection avoidance and the primary violation. More generally, suppose we raise the uniform sanction from some existing level. An application of the envelope theorem to (1) (using logic similar to that used in deriving (2)) establishes that raising the uniform sanction always increases primary activity deterrence Δ . However, as we will show, it does so at the cost of increasing detection avoidance. Increasing the uniform sanction can only increase those orders of detection avoidance that the violator currently sets at 0. For all orders $i = 1, 2, \dots$ for which the violator is currently exercising positive detection avoidance, the violator is setting:

$$-\sum_{j=0}^{i-1} \frac{\delta p_j}{\delta a_i} s = \frac{\delta p_i}{\delta a_i} s + 1 \Leftrightarrow \left(\underbrace{-\sum_{j=0}^{i-1} \frac{\delta p_j}{\delta a_i}}_{+} - \underbrace{\frac{\delta p_i}{\delta a_i}}_{+} \right) s = 1 \quad (3)$$

The first term in parentheses is order i 's sanctions-reducing upside. The second, its sanctions-increasing downside. To satisfy (3), the net of these two terms must be positive, so that (multiplied by the sanction s) their sum equals the direct marginal cost of i^{th} order detection avoidance. Therefore, increasing s increases the net marginal upside of i^{th} order detection avoidance and thereby increases i^{th} order detection avoidance. Given our assumptions on p_i , this increases a_i . (The parenthesis in (3) is the net marginal benefit of a_i ; if this increases due to a change in s , the violator responds by increasing a_i until the marginal benefit diminishes back down to the fixed marginal cost of 1.) More formally, the implicit function theorem yields:

$$\frac{\delta a_i}{\delta s} = - \left(\underbrace{-\sum_{j=0}^{i-1} \frac{\delta p_j}{\delta a_i} - \frac{\delta p_i}{\delta a_i}}_{+} \right) / \left(\underbrace{-\sum_{j=0}^{i-1} \frac{\delta^2 p_j}{\delta a_i \delta a_i} - \frac{\delta^2 p_i}{\delta a_i \delta a_i}}_{-} \right) s > 0.$$

5. Sanctioning hierarchies: variable

For sanctions that vary across orders, we have already seen that the envelope theorem implies that increasing the sanction *at any given*

order increases primary deterrence. Therefore, an increase in all sanctions, possibly varying in magnitude across orders, increases primary deterrence. What kind variation in increase across orders also reduces detection avoidance spending? Focus on a particular order i . Recall that

$$\sum_{j=0}^{i-1} \underbrace{\left(-\frac{\delta p_j}{\delta a_i} \right)}_{+} s_j - \underbrace{\frac{\delta p_i}{\delta a_i}}_{+} s_i = 1. \quad (4)$$

Our assumptions on p_i imply that the left hand side decreases in a_i . The implicit function theorem, therefore, implies that any change in the sequence s_0, s_1, \dots that increases (decreases) the left side—evaluated at the violator's current choice of a_i —will cause the violator to increase (decrease) a_i . The question, then, is what change in the sequence of sanctions decreases the left side? The condition for decrease is:

$$\sum_{j=0}^{i-1} \underbrace{\left(-\frac{\delta p_j}{\delta a_i} \right)}_{+} ds_j - \underbrace{\frac{\delta p_i}{\delta a_i}}_{+} ds_i < 0 \quad (5)$$

Combining (5) with the first order condition (4) we have:

$$\frac{\sum_{j=0}^{i-1} \left(-\frac{\delta p_j}{\delta a_i} \right) ds_j}{\sum_{j=0}^{i-1} \left(-\frac{\delta p_j}{\delta a_i} \right) s_j} < \frac{ds_i}{s_i} \Leftrightarrow \sum_{j=0}^{i-1} \alpha_j \frac{ds_j}{s_j} < \frac{ds_i}{s_i}, \text{ where } \alpha_j = \frac{\left(-\frac{\delta p_j}{\delta a_i} s_j \right)}{\sum_{j=0}^{i-1} \left(-\frac{\delta p_j}{\delta a_i} \right) s_j}. \quad (6)$$

In words, a necessary condition for i^{th} order detection avoidance to decrease is that the percentage change in s_i exceed the (weighted) average percentage change in lower order s_j . (In the case in which a_i affects only p_{i-1} , (6) reduces to $\frac{ds_{i-1}}{s_{i-1}} < \frac{ds_i}{s_i}$.) Thus, if we are increasing all lower order sanctions by 10%, the sanction at order i must be increased by more than 10%. This continues to be so even after the base for the higher sanction has grown much larger than that for lower order sanctions.

Conversely, increasing the higher order sanction by a *lower* proportion than the lower causes the left side of (4) to increase even more than it would were all sanctions increased by the same proportion, as in the uniform sanction case examined above.

6. The “technological” attack

Setting sanctions for all orders of detection avoidance equal to zero, focus on the effect of d_0 on $p_0(d_0, a_1, a_2, \dots)$. For purposes of this analysis, we may combine detection avoidance into one variable a and we may drop all “0” subscripts. The violator sets $\frac{\delta p}{\delta a}(d, a)_s = -1$. In Figure 1 this is where detection avoidance is a_0 and the probability of detection is p_0 , the point at which the slope of the detection probability curve ($\frac{\delta p}{\delta a}(d, a)_s$) equals the slope (-1) of the 45 degree line.

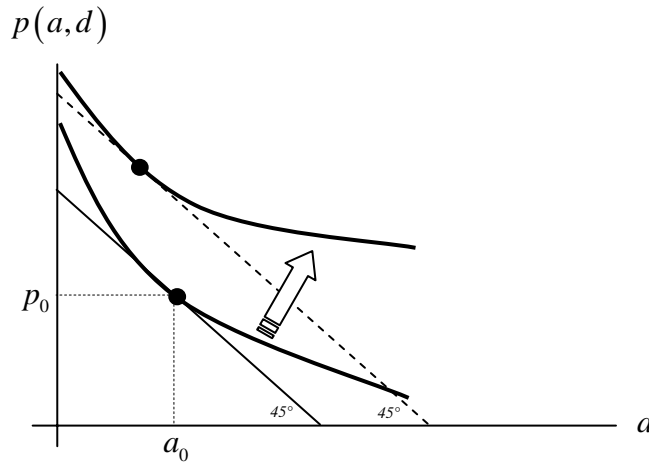


FIGURE 1: Intensifying the technological attack flattens the detection probability curve

An intensification of the technological attack is an increase in the parameter d that: 1) reduces the marginal productivity of detection avoidance, i.e., $-\frac{\delta^2 p}{\delta a \delta d}(d, a) < 0$, and 2) does not decrease the probability of detection in the absence of detection avoidance, i.e., $\frac{\partial p}{\partial d}(0, d) \geq 0$. In other words, an intensification of the technological approach flattens (and does not shift down) the probability curve, as in Figure 1. In that case

$$\frac{\delta a}{\delta d} = \frac{\overbrace{-\frac{\delta^2 p}{\delta a \delta d}(d, a)}^{\text{flattening}}}{\underbrace{-\frac{\delta^2 p}{\delta a \delta a}(d, a)}_{\text{negative}}} < 0,$$

so that detection avoidance spending decreases, as shown in Figure 1. For the effect on deterrence, first note that the probability of

detection must increase, $\frac{\partial p}{\partial d}(a, d) = \int_0^a \overbrace{\frac{\partial^2 p}{\partial a \partial d}}^+ (a, d) + \overbrace{\frac{\partial p}{\partial d}}^+(0, d) > 0$, as also shown in Figure 1. Therefore, raising d increases deterrence:

$\frac{\delta \Delta}{\delta d} = \overbrace{\frac{\delta p}{\delta d}}^+ s > 0$. The marginal deterrence cost of d becomes:

$$\frac{\delta C}{\delta d} / \frac{\delta \Delta}{\delta d} = (1 - F(\bar{\Delta})) \left(1 + \overbrace{\frac{\delta a}{\delta d}}^- \right) / \overbrace{\frac{\delta p}{\delta d}}^+ s.$$

Increasing d is more cost effective the more doing so reduces the productivity of detection avoidance. Indeed, it is even possible that increasing d will produce more deterrence at lower social cost. Such is the case if $\delta a / \delta d < -1$.

Reducing the marginal productivity of detection avoidance is not the same as increasing the marginal cost of probability reductions. This is an important distinction because a marginal cost increase does not necessarily lead to a reduction in detection avoidance spending: although the violator “buys” fewer probability points, each costs more.

Given $p_a < 0$ and $p_{aa} > 0$, a marginal productivity reduction implies, but is not implied by a marginal cost increase. Marginal cost is $-\frac{\partial a}{\partial p}(p, d) = \left(-p_a(p^{-1}(p, d), d) \right)^{-1}$. This increases in d if and only if $-p_a(p^{-1}(p, d), d)$ decreases in d . Taking the derivative of this expression yields

$$\frac{-\partial p_a(p^{-1}(p, d), d)}{\partial d} = -\overbrace{\frac{\partial^2 p}{\partial a \partial a}}^+ \overbrace{\frac{\partial a}{\partial d}}^+(p, d) - \frac{\partial^2 p}{\partial a \partial d},$$

where $p_a < 0$ and $\frac{\partial p}{\partial d} > 0$ (derived above) implies $\frac{\partial a}{\partial d} > 0$. Therefore, $\frac{\partial^2 p}{\partial a \partial d} > 0$ is sufficient for increasing marginal cost, but marginal cost can decrease even if $\frac{\partial^2 p}{\partial a \partial d} < 0$. For a specific example of the latter possibility, consider $p(a, d) = e^{-a/d} \Leftrightarrow a(p, d) = -d \ln p$, wherein $-a_{pd} = \frac{1}{p} > 0$, so that marginal cost increases in d , but $-p_{ad} = \frac{p}{d^2} \left(\frac{a}{d} - 1 \right)$, so that marginal productivity goes down in d only for low a .

